**Homework 6 Solutions**

**Problem 1.** Two strings are connected together, both under tension T = 80N. The first has mass density μ1 = 10g/m, and the second has mass density μ2 = 50g/m. A wave pulse with amplitude A1 = 6cm and frequency f1 = 30Hz is sent down the first string. What are:

(a) speed of incident wave? √T/μ = √80/0.010 = 89 m/s

(b) wavelength of incident wave? λ = v/f = 73/30 = 2.97m

(c) energy in wavelength of incident wave? Eiλ = (1/2)μ(Aω)2λ = 1.9J

(d) amplitude of reflected wave? And is it inverted? Ar = (n1 - n2)/(n1 + n2) = -2.3cm. Yes.

(e) speed of reflected wave? vr = 89 m/s

(f) frequency of reflected wave? fr = 30Hz

(g) wavelength of reflected wave? λr = 2.97m

(h) energy in wavelength of reflected wave? Erλ = (1/2)μ(Aω)2λ = 0.28J

(i) amplitude of transmitted wave? And is it inverted? At = 2n1 /(n1 + n2) = 3.7cm, nope.

(j) speed of transmitted wave? vt = √T/μ = 40m/s

(k) frequency of transmitted wave? ft = 30Hz

(l) wavelength of transmitted wave? λt = vt/ft = 1.33m

(m) energy in wavelength of transmitted wave? Etλ = 1.62J

(n) is energy conserved between the incident, transmitted, and reflected waves? Yup.

**Problem 2.** The speed of sound in air is 345m/s, and its speed in water is 1480m/s. Now suppose you’re in the water, and your Mom is shouting directly above you (in the air) to do the dishes. Model the sound wave as plane wave (then it can be treated in 1D fashion) so that we can say u = (1/2)μ(Aω)2, and n = √μ. Using only the numbers in the problem statement, answer the following:

(a) What fraction of the incident wave’s amplitude will the transmitted wave’s amplitude be?

So,



(note we’re using n1v1 = n2v2 → n2/n1 = v1/v2)

(b) What fraction of the reflected wave’s amplitude will the reflected wave’s amplitude be?

And,



(c) What fraction of the incident wave’s intensity will the transmitted wave have?

Well, since you ask,



(d) What fraction of the incident wave’s intensity will the reflected wave have?

Don’t mind if I do,



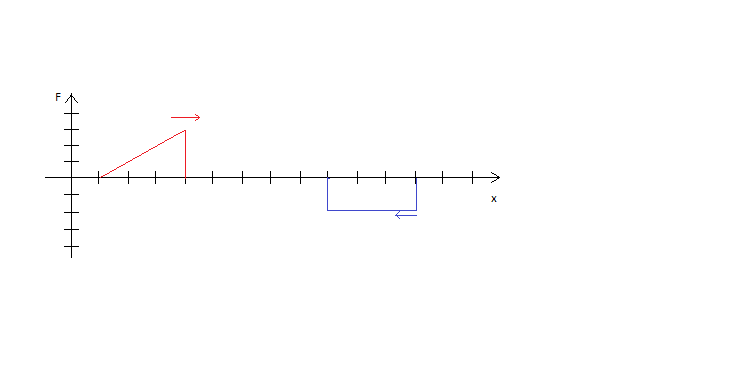
(e) If your Mom’s sound wave has a volume β = 50dB, then what will be the volume of the wave you hear?

So,

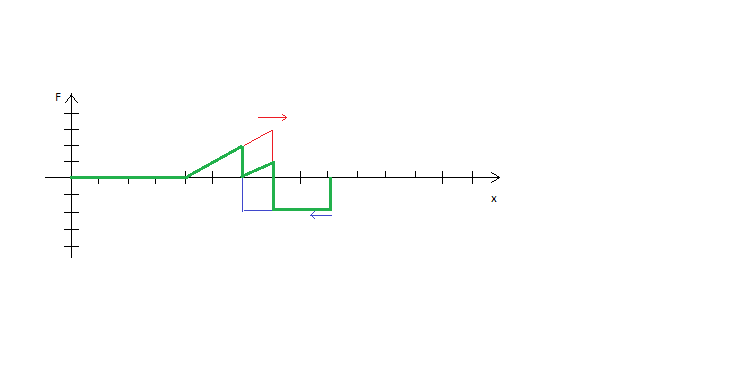


So you can’t say you didn’t hear her, though maybe you didn’t understand her.

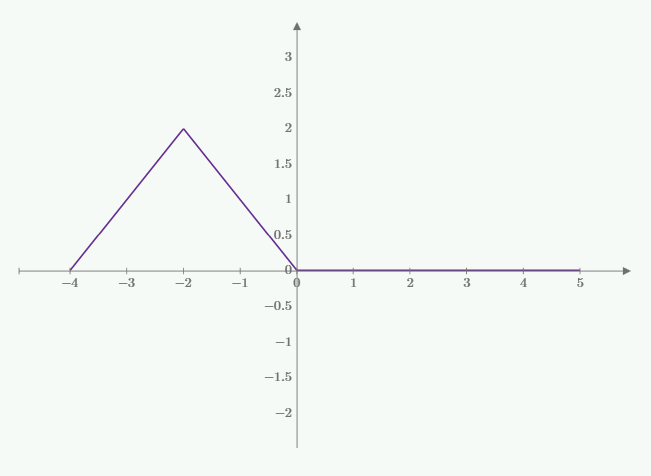
**Problem 3.** The Emperor and Yoda both send force pulses towards each other, as shown below. What will the net force look like in 3s? Each pulse travels at a speed of 1 unit/s.



It will look like, in bold green,

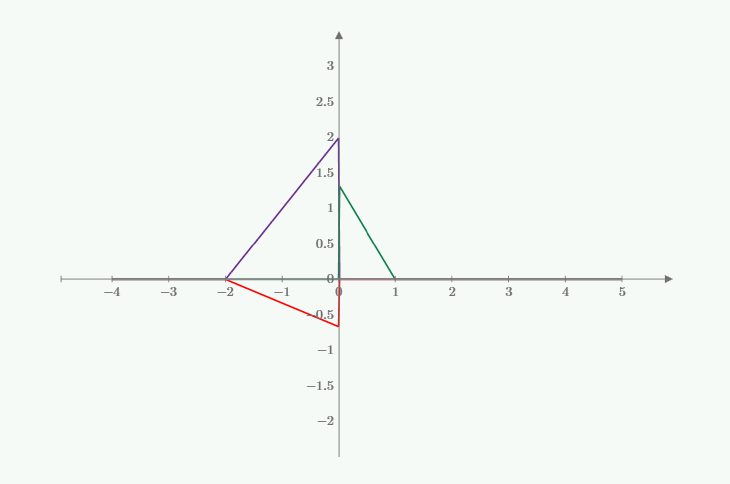


**Problem 4.** For your consideration, I offer a sound wave traveling with velocity v = 1 unit/second, down a medium (x<0) with index of refraction n = 1. It’s about to hit a medium (x>0) with index of refraction n = 2.



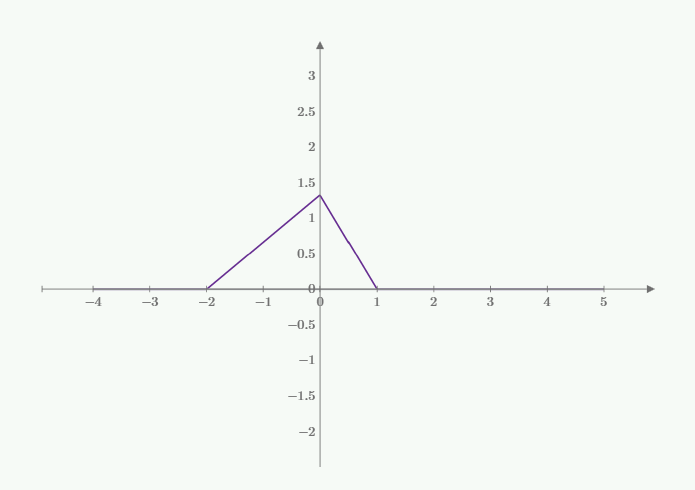
(a) What will the incident, transmitted, and reflected waves look like in 2s?

The velocity of the incident and reflected waves will be v1 = 1unit/s. The velocity of the transmitted wave will be: n1v1 = n2v2 → v2 = (n1/n2)v1 = (1/2)(1) = 0.5 unit/s. Also, the amplitudes of the waves will be: Ar = (n1 – n2)/(n1 + n2)∙Ai = (1-2)(1+2)∙Ai = (-1/3)Ai, and At = 2n1/(n1+n2)Ai = 2(1)/(1+2)Ai = 0.67Ai. So in two seconds, the incident wave will have progressed a distance (1 unit/s)(2s) = 2units. So we translate it over this distance. Then reflect the part that’s past x = 0 back onto the the -x axis region, but upside down and with 1/3 the height. Then take the transmitted part and reduce its height to 2/3 the original, and compress it back to x = 1 since it will have only traveled 1 unit in the 2s interval, since its speed is 0.5 unit/s.



(b) What will the total wave look like in 2s?

Just add the coordinates of the incident and reflected waves, wherever they overlap (which is everywhere in this case).

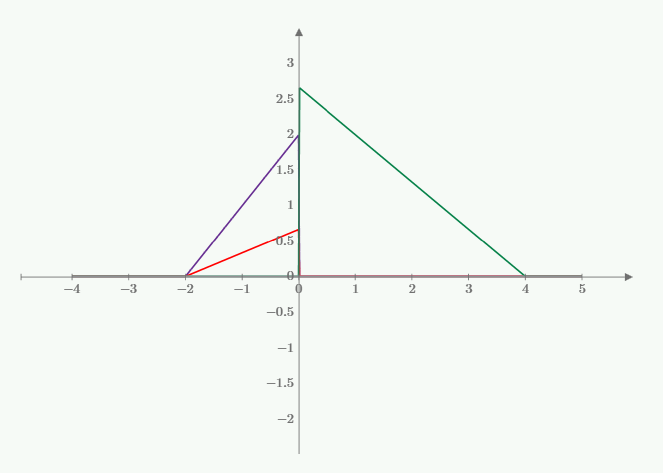


**Problem 5.** Now, since that was so much fun, I offer the same sound wave traveling down medium (x<0) with velocity v = 1unit/second. But now medium 1 has index of refraction n = 2, and medium 2 (x>0) has index of refraction n = 1.

(a) What will the incident, transmitted, and reflected sound waves look like in 2s?

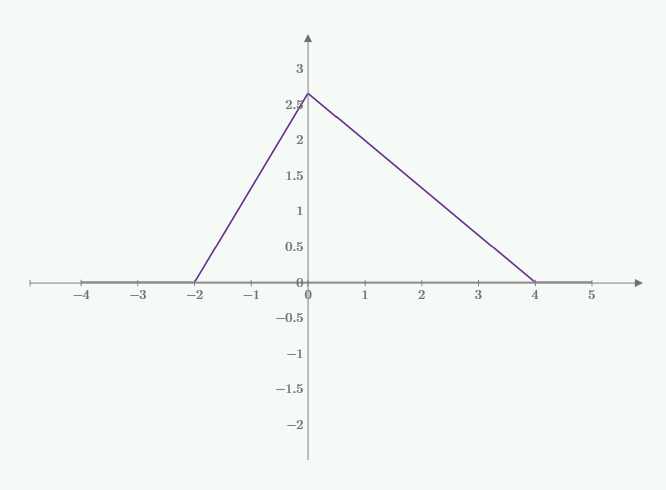
Same as above, except now speed in medium 2 is v2 = (n2/n1)v1 = (2/1)(1) = 2unit/s. And amplitudes are:

Ar = (2-1)/(2+1)Ai = (1/3)Ai, and At = 2(2)/(2+3) = 4/3. So again, we translate the incident wave forward two units, reflect the transmitted part, without inverting it this time, and with (1/3) height. And then augment the height of the transmitted part to 4/3 original, and stretch it out to x = 4, since its velocity is v = 2 unit/s, rather than 1 unit/s.

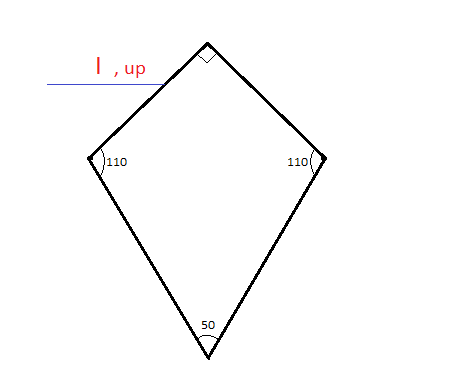


(b) And what will the total wave look like?

Now add up the displacements of the incident and reflected waves, where the overlap (everywhere again).



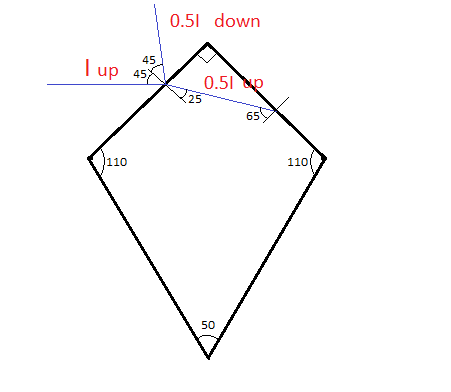
**Problem 6.** One languid morning, you’re shopping online for a diamond for your girlfriend/boyfriend/whateverfriend. A particularly sparkly picture captures your eye, and evokes fond memories of your physics class. Suddenly a profound desire to trace the path of a light ray through this diamond overcomes your lassitude. So you shine a laser pointer with intensity I, through the diamond (with index of refraction 1.67), as shown. Sketch the path of all reflected and refracted (transmitted) rays up to 4 reflections. You may recall that we said it was too complicated to work out the amplitudes/intensities of the transmitted/reflected rays for incidence angles not equal to 0. Nonetheless, just suppose that each time the beam splits, half the intensity goes to the reflected ray, and half goes to the transmitted ray, and so label each ray’s intensity as a fraction of I. Oh and I had another idea: suppose the incident ray’s displacement is ‘up’. State the orientation of each transmitted and reflected ray as well (either up or down).



Oh such a beautiful diamond! The angle of incidence is 45°, and of course the angle of reflection will be as well. Then using Snell’s law to get the first angle of incidence we have:



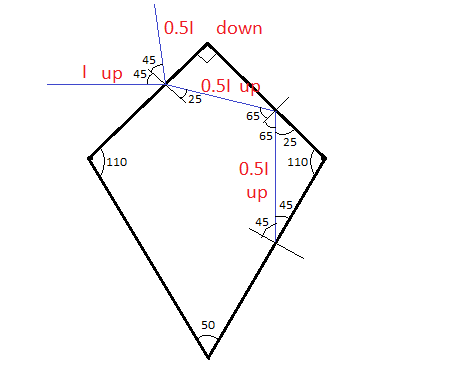
So



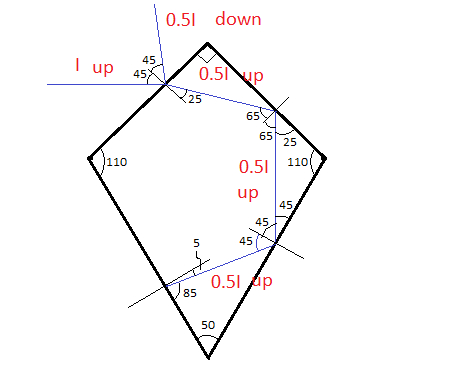
Again, the reflected ray will make a 65° as well. And for the refracted ray we have:



which is not a *real* number (your calculator might simply say, undefined). This is an indication that the angle of incidence is past the critical angle. Indeed, the critical angle is: θc = sin-1(1/1.67) = 37°. And so we conclude that the light ray *only* reflects off of the surface. So then we have:



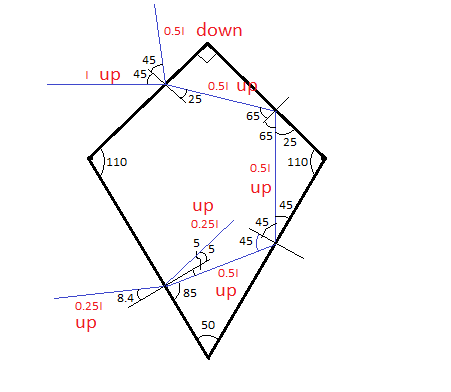
The next angle of incidence is 45°. But this angle of incidence is also greater than θc. So it will *only* reflect off of this surface as well. So then we have:



Now the angle of incidence is less than θc, so it will both reflect and transmit. Angle of reflection is 5° as well, and the angle of transmission is:

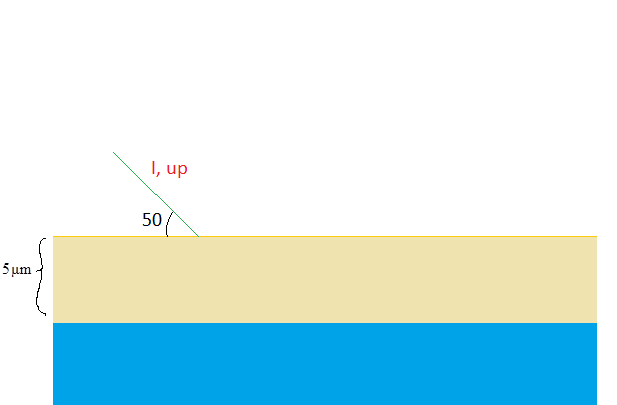


So,

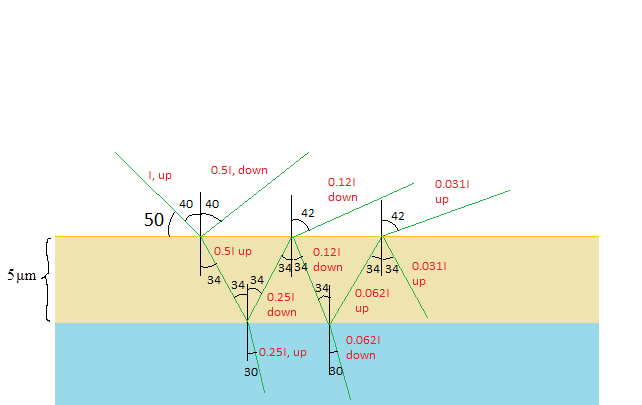


**Problem 7.** Driving over to your friends house, to work on the homework, you stop for gas. When you put the pump back up, you drop a little gas on a water puddle. Dazzled by the swirl of colors, you embark upon another intrepid physics analysis. The index of refraction of oil is 1.2, and that of water is 1.33. The oil film is 5μm thick. You shine a 500nm wavelength light wave into the medium.

(a) Draw the path of the light ray (intensity I, orientation ‘up’), up to five reflections. Assuming the light splits its energy evenly between the reflected and transmitted rays, give the intensity of each ray. And also give the orientation of each ray.



Well, looks like this:



(b) What is the speed of the light ray in the air? In the oil? In the water?

Speed of light in air is c = 3×108m/s. Then, using n1v1 = n2v2, we get:



(c) What is the wavelength of the light wave in air? In the oil? In the water?

The wavelength changes through the medium in the same way the speed does: n1λ1 = n2λ2. So,



(c) What is the frequency of the light ray in the air? In the oil? In the water?

These are all the same, as frequency never changes: f1 = f2. As you can see,



(d) What is the transit time between when the light ray first enters the oil, and when it (or part of it rather) first exits?

Transit time is:

